Preface

Special Issue on Polynomial and Tensor Optimization

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Polynomial and tensor optimization has been an active area of research in the last decade, primarily driven by the recently found wide range of applications in computer vision, image and signal processing, psychometrics, chemometrics, quantum physics, machine learning and statistics, among many others. The surge of research interest has been echoed and supported by a number of path-breaking developments on the theoretical side as well. Noticeable discoveries along this line include: polynomial minimization via the sum-of-squares polynomial approximation which leads to semidefinite programming; the link between symmetric tensor approximation and polynomial optimization; extension of compressive sensing to the tensor format; the newly discovered properties of various tensor ranks and eigenvalues. This volume aims to solicit some of the timely results on polynomial and tensor optimization in a broadly defined sense around the above-mentioned themes. Through a rigorous peer review process, 11 papers are finally accepted to be included in this special volume. These papers can be classified into the following three main categories.

The first category focuses on polynomial optimization models. Amaral and Bomze propose a copositive reformulation of the mixed-integer fractional quadratic problem under general linear constraints. Results in this paper show evidence for the applicability of copositive optimization approaches and provide novel approximation strategies in the domain of (fractional) polynomial optimization. Two papers discuss approximation algorithms for some important classes of polynomial optimization models. In particular, Xu and Hong propose a semidefinite programming relaxation and randomization technique for an NP-hard mixed binary quadratically constrained quadratic program and analyze its approximation performance. The problem is to find two minimum norm vectors in n-dimensional real or complex Euclidean space, such that half of the concave quadratic functions are satisfied. Ling, Zhang and Qi study the problem of minimizing a multi-quadratic form over the Cartesian product of several simplices. Several lower bounding techniques for this problem ranging from very simple and cheap ones to more complex constructions are presented, and a related bilinear copositive programming reformulation is presented.

Relating polynomial optimization to tensor problems, the second category includes the computation of tensor eigenvalues and rank-one approximation of tensors. Zeng and Ni propose a quasi-Newton method for computing Z-eigenpairs of a symmetric tensor. The iterative sequence generated by the quasi-Newton method is norm descent for the function corresponding to the eigenvalue equations. The global and superlinear convergence of the proposed method is established. Hao, Cui and Dai propose a feasible trust-region method for calculating extreme Z-eigenvalues of symmetric tensors. Global convergence and local quadratic convergence of the proposed method are established. Uschmajew studies the convergence of higher-order power method for computing the extreme singular values of tensors. The convergence proof is based on the so-called Lojasiewicz inequalities to the equivalent, unconstrained alternating least squares algorithm for best rank-one tensor approximation. Kong and Meng discuss the bounds for the best rank-one approximation ratio of a finite dimensional tensor space. In particular, they propose exact values for the best rank-one approximation ratio of third order tensors when the smallest dimension of the three modes is 2. They also show that the exact value of the best rank-one approximation ratio of a real tensor space tends to be different with the ratio of the complex tensor space with the same dimension.

The last category includes tensor recovering and completion problems through low rank approximation. Two papers are on the theoretical side of this study. Huang, Mu, Goldfarb and Wright rigorously study tractable models for provably recovering low-rank tensors. They propose a class of convex recovery models that are shown to guarantee exact recovery under a set of new tensor incoherence conditions. Zhang and Huang investigated the conditions that guarantee the equivalence between the low-n-rank tensor recovery and its tractable convex relaxation problems. They discuss three conditions for the equivalence, which are the null space property, the s-n-good condition and the restricted isometry property, and they also discuss the relationship among these three conditions. In terms of algorithms, Gao, Jiang and Tao propose a new method for finding decompositions of a low-CP-rank tensor. Block coordinate descent method is applied to solve the resulting optimization problem and numerical results are shown for tensor completion problems. On the application side, Xiu and Kong study tensor optimization in surveillance video. They consider a rank-min-one and sparse tensor decomposition model for surveillance video and propose a modified iterative reweighted algorithm. The numerical experiments illustrate the efficiency of the proposed algorithm.

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